3) $\frac{d}{dx}(3x^8+2x+1)$

$= 3\frac{d}{dx}(x^8) + 2\frac{d}{dx}(x^1) + \frac{d}{dx}(x^0)$

$= 3(8x^7) + 2(x^0) + (0x^{-1})$

$= 24x^7 + 2$

$y = 1$

$\frac{dy}{dx} = ?$

Derivative
fn
(inst) rate of chg
$\Rightarrow$ slope of tangent

$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

5) $y=\pi^3$ $=0$

[no Xs]

6) $y=\sqrt{2}x+\dfrac{1}{\sqrt{2}}$

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{2}x\right)+\frac{d}{dx}\left(\frac{1}{\sqrt{2}}\right)$$

$$=\sqrt{2}\left(\frac{d}{dx}(x)\right)+\frac{d}{dx}\left(\frac{1}{\sqrt{2}}\right)$$

$$=\sqrt{2}(1)=\sqrt{2}$$

6
5
8
10
11
12

8) $y = \dfrac{x^2+1}{5} = \dfrac{x^2}{5} + \dfrac{1}{5}$

$$\frac{dy}{dx} = \frac{1}{5}\frac{d}{dx}(x^2) + \frac{d}{dx}\left(\frac{1}{5}\right)$$

$$= \frac{1}{5}(2x) = \frac{2}{5}x$$

10) $y = \frac{1}{a}\left(x^2 + \frac{1}{b}x + c\right) = \frac{1}{a}x^2 + \frac{1}{ab}x + \frac{c}{a}$

$$\frac{dy}{dx} = \frac{1}{a}\frac{d}{dx}(x^2) + \frac{1}{ab}\left(\frac{d}{dx}(x)\right) + \frac{d}{dx}\left(\frac{c}{a}\right)$$

$$= \frac{2x}{a} + \frac{1}{ab} = \frac{1}{a}\left(2x + \frac{1}{b}\right)$$

$$\frac{d}{dx} = \frac{1}{a}\left(\frac{d}{dx}\left(x^2 + \frac{1}{b}x + c\right)\right) = \frac{1}{a}\left(2x + \frac{1}{b}\right)$$

11) $y = -3x^{-8} + 2\sqrt{x}$

$= -3x^{-8} + 2x^{1/2}$

$\frac{dy}{dx} = 24x^{-9} + x^{-1/2}$ $\frac{1}{x^{1/2}}$

$= 24x^{-9} + \frac{1}{\sqrt{x}}$

$= \frac{24}{x^9} + \frac{1}{\sqrt{x}}$

12) $y = 7x^{-6} - 5\sqrt{x}$

$= 7x^{-6} - 5x^{1/2}$

$= -42x^{-7} - \frac{5}{2}x^{-1/2}$

13) $f(x) = x^{-3} + \frac{1}{x^7}$

$= \frac{1}{x^3} + \frac{1}{x^7}$

18
20
13
16
17

$f'(x) = \frac{(0)(x^3) - (1)(3x^2)}{(x^3)^2} + \frac{(0)(x^7) - (1)(7x^6)}{(x^7)^2}$

$= \frac{-3x^2}{x^6} + \frac{-7x^6}{x^{14}}$

$= \frac{-3}{x^4} - \frac{7}{x^8}$

$= -3x^{-4} - 7x^{-8}$

16) $f(x) = (2 - x - 3x^3)(7 + x^5)$

$f'(x) = f'g + fg'$

$= (-1 - 9x^2)(7 + x^5) + (2 - x - 3x^3)(5x^4)$

$= -7 - x^5 - 63x^2 - 9x^7 + 10x^4 - 5x^5 - 15x^7$

$= -24x^7 - 6x^5 + 10x^4 - 63x^2 - 7$

17) $f(x) = (x^3 + 7x^2 - 8)(2x^{-3} + x^{-4})$

$f'(x) = f'g + fg'$

$= (3x^2 + 14x)(2x^{-3} + x^{-4}) + (x^3 + 7x^2 - 8)(-6x^{-4} - 4x^{-5})$

$= \cancel{6}x^{-1} + 28x^{-2} + 3x^{-2} + 14x^{-3} +$

$-\cancel{6}x^{-1} - 4x^{-2} - 42x^{-2} - 28x^{-3} + 48x^{-4} + 32x^{-5}$

$-15x^{-2} - 14x^{-3} + 48x^{-4} + 32x^{-5}$

18) $f(x) = \left(\frac{1}{x} + \frac{1}{x^2}\right)(3x^3 + 27)$

$x^{-1} + x^{-2}$

$f'(x) = \overset{f'}{(-x^{-2} - 2x^{-3})}\overset{g}{(3x^3 + 27)} + \overset{f}{(x^{-1} + x^{-2})}\overset{g'}{(9x^2)}$

$= -3x - 27x^{-2} - 6 - 54x^{-3} + 9x + 9$

$= 6x + 3 - 27x^{-2} - 54x^{-3}$

20) $f(x) = (x^5+2x)^2 = (x^5+2x)(x^5+2x)$

$f'(x) = (5x^4+2)(x^5+2x) + (x^5+2x)(5x^4+2)$

$= 2(5x^9+10x^5+2x^5+4x)$

$= 10x^9+24x^5+8x$

21) $y = \frac{1}{5x-3}$; find $y'(1)$

$\frac{d}{dx}\left(\frac{1}{x}\right)$ uses power rule

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$$y' = \frac{\cancel{\frac{d}{dx}(1)(5x-3)} - (1)\frac{d}{dx}(5x-3)}{(5x-3)^2}$$

$$= \frac{-(5)}{(5x-3)^2}$$

$$y'(1) = \frac{-5}{(5(1)-3)^2} = \frac{-5}{4}$$

22) $y = \frac{3}{\sqrt{x}+2}$; $y'(1) = ?$

21, 22
27, 28
34, 33

$$y' = \frac{\cancel{\frac{d}{dx}(3)(\sqrt{x}+2)} - (3)\frac{d}{dx}(\sqrt{x}+2)}{(\sqrt{x}+2)^2}$$

$$= \frac{-3\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x}+2)^2}$$

$\frac{d}{dx}(\sqrt{x}+2)$
$= \frac{d}{dx}(\sqrt{x})$
$= \frac{d}{dx}(x^{1/2})$
$= \frac{1}{2}x^{-\frac{1}{2}}$

$$y'(1) = \frac{-3\left(\frac{1}{2\sqrt{1}}\right)}{(\sqrt{1}+2)^2}$$

$$= \frac{-3\left(\frac{1}{2}\right)}{9} = -\frac{1}{6}$$

3.3 homework 2010-10-04 Pd 2

27) $y=\left(\frac{3x+2}{x}\right)(x^{-5}+1)$ find $\left.\frac{dy}{dx}\right|_{x=1}$

$(fg)'=f'g+fg'$

$f=\left(\frac{3x+2}{x}\right)$; $f'=\frac{\frac{d}{dx}(3x+2)(x)-(3x+2)\frac{d}{dx}(x)}{(x)^2}=\frac{(3)x-(3x+2)(1)}{x^2}=\frac{-2}{x^2}$

$g=(x^{-5}+1)$; $g'=-5x^{-6}$

$y'=\left(-\frac{2}{x^2}\right)(x^{-5}+1)+\left(\frac{3x+2}{x}\right)(-5x^{-6})$

$=\left(-\frac{2}{x^2}\right)\left(\frac{1}{x^5}+1\right)+\left(\frac{3x+2}{x}\right)\left(\frac{-5}{x^6}\right)$

$y'(x=1)=\frac{-2}{1}\left(\frac{1}{1}+1\right)+\left(\frac{5}{1}\right)\left(\frac{-5}{1}\right)=-4-25=-29$

3.3 homework

27) $y=\left(\frac{3x+2}{x}\right)(x^{-5}+1)=\left(\frac{3x}{x}+\frac{2}{x}\right)(x^{-5}+1)=(3+2x^{-1})($

$y'=\frac{d}{dx}(3+2x^{-1})(x^{-5}+1)+(3+2x^{-1})\left(\frac{d}{dx}(x^{-5}+1)\right)$

$=(-2x^{-2})(x^{-5}+1)+(3+2x^{-1})(-5x^{-6})$

$y'(1)=(-2)(2)+(5)(-5)=-29$

28) $y = (2x^7 - x^2)\left(\frac{x-1}{x+1}\right)$

$$y' = (14x^6 - 2x)\left(\frac{x-1}{x+1}\right) + (2x^7 - x^2)\left(\frac{(1)(x+1) - (x-1)(1)}{(x+1)^2}\right)$$

$$= (14x^6 - 2x)\left(\frac{x-1}{x+1}\right) + (2x^7 - x^2)\left(\frac{2}{(x+1)^2}\right)$$

$$y'(1) = (12)(0) + (1)\left(\frac{1}{2}\right) = \frac{1}{2}$$

$\pi \approx 3.14159\ldots$

3.142 , 3.141

33) $\frac{d}{dt}(16t^2) = 16 \cdot 2 \cdot t^{2-1} = 32t$

34) $\frac{dC}{dr}$; where $C = 2\pi r$

$\frac{d(2\pi r)}{dr} = (2\pi)\frac{d}{dr}(r) = 2\pi(1) = 2\pi$

39) Find $g'(4)$ given $f(4)=3$ and $f'(4)=-5$

a) $g(x) = (\sqrt{x})(f(x))$

$\sqrt{x} = x^{1/2}$

drv: $\frac{1}{2}x^{-\frac{1}{2}}$

$$g'(x) = (\text{first})'(\text{second}) + (\text{first})(\text{second})'$$

$$g'(x) = \left(\frac{1}{2\sqrt{x}}\right) f(x) + \sqrt{x}\, f'(x)$$

$$g'(4) = \frac{1}{2\sqrt{4}} f(4) + \sqrt{4}\, f'(4)$$

$$= \frac{1}{4}(3) + 2(-5) = \frac{3}{4} - 10 = \frac{-37}{4}$$

b) $g(x) = \frac{f(x)}{x}$

$$g'(x) = \frac{(\text{top})'(\text{bottom}) - (\text{top})(\text{bottom})'}{(x)^2}$$

$$= \frac{(f'(x))(x) - (f(x))(1)}{x^2}$$

$$g'(4) = \frac{(f'(4))(4) - f(4)}{4^2} = \frac{(-5)(4) - 3}{16}$$

$$= \frac{-23}{16}$$

40) $g'(3)$ given $f(3) = -2$; $f'(3) = 4$

b) $g(x) = \dfrac{2x+1}{f(x)}$

$$g'(x) = \frac{(2)(f(x)) - (2x+1)(f'(x))}{(f(x))^2}$$

$$g'(3) = \frac{2f(3) - (2(3)+1)f'(3)}{(f(3))^2}$$

98
12T

$$= \frac{2(-2)-(7)(4)}{(-2)^2} = \frac{-4-28}{4} = -8$$

44) Find eqn for tangent line to curve $y = f(x) = \frac{(1-x)}{(1+x)}$ at $x=2$.

To write the eqn of a line:

☆ slope

$= f'(2)$

$f'(x) = \frac{(-1)(1+x)-(1-x)(1)}{(1+x)^2}$

$f'(2) = \frac{(-1)(1+2)-(1-2)(1)}{(1+2)^2}$

$= \frac{-3+1}{9} = -\frac{2}{9}$

☆ a point

$(2, f(2)) =$

$\left(2, \frac{1-2}{1+2}\right) = \left(2, -\frac{1}{3}\right)$

Pt - slope form

slope = m

pt = (x_0, y_0)

then eqn of a line

$y - y_0 = m(x - x_0)$

Consider

$m = \frac{y-y_0}{x-x_0}$

write an eqn:

"PT SLOPE"

$y - \left(-\frac{1}{3}\right) = -\frac{2}{9}(x-2)$

42) Find $F'(\pi)$ given $f(\pi)=10$, $f'(\pi)=-1$

$g(\pi)=-3$, $g'(\pi)=2$

c) $F(x)=(2)(f(x))(g(x))$

$F'(x)=2\,\frac{d}{dx}\left(f(x)\,g(x)\right)$

$=2\left[f'(x)g(x)+f(x)g'(x)\right]$

$F'(\pi)=2\left[f'(\pi)g(\pi)+f(\pi)g'(\pi)\right]$

$=2\left[(-1)(-3)+(10)(2)\right]$

$=2\left[+3+20\right]$

$=46$

d) $F(x)=\dfrac{f(x)}{4+g(x)}$

$$F'(x)=\frac{f'(x)(4+g(x))-f(x)(g'(x))}{(4+g(x))^2}$$

$$F'(\pi)=\frac{f'(\pi)(4+g(\pi))-f(\pi)(g'(\pi))}{(4+g(\pi))^2}$$

$$=\frac{(-1)(4+-3)-(10)(2)}{(4+(-3))^2}=-1-20=-21$$

57) Find f^n $y=ax^2+bx+c$ w/

① → x-intercept of 1

② → y-intercept of 2

③ → tangent line, $m=-1$ at y-intercept

$x^2-dx-1x+d$

$ax^2+bx+c=a(x-1)(x-d)$ ①

$=a(x^2-(d+1)x+d) = ax^2-a(d+1)x+ad$

$ad=2$ ②

③ → slope of tan = derivative $= 2ax-a(d+1) = -1$ when $x=0$

$-a(d+1)=-1$ ③

$ad=2$

$d=\frac{2}{a} \Rightarrow -a\left(\frac{2}{a}+1\right)=-1$

$a\left(\frac{2}{a}+1\right)=+1$

$2+a=1$

$a=-1$ ✓

$d=\frac{2}{(-1)}=-2$

Polynomial $=$

$a(x-1)(x-d)$

$=(-1)(x-1)(x-{-2})$

$=(-1)(x-1)(x+2)$

76) $f(x) = \begin{cases} x^2 - 16x, & x < 9 \\ 12x^{1/2}, & x \ge 9 \end{cases}$

Cont @ $x = 9$?

(b) $f(9)$ exists; $f(9) = 12\sqrt{9} = 36$

(a) $\lim_{x \to 9^+} f(x) = \lim_{x \to 9^+} 12\sqrt{x} = 36$

$\lim_{x \to 9^-} f(x) = \lim_{x \to 9^-} x^2 - 16x = 81 - 144 = -63$

∴ function is NOT Continuous, (b/c 2 sided limit DNE)

If f is differentiable [i.e. derivative exists]

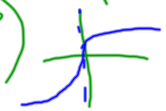
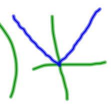
If f is a square then f is also a rectangle. $f_{\text{square}} \Rightarrow f_{\text{rectang}}$

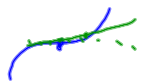
cont

diff

then f is also continuous

Be careful! There ARE continuous functions that are NOT differentiable.

Ex: $(y = x^{1/3})$  ; $(y = |x|)$ 



59) Find the x-coordinate of the pt on the graph of $y=x^2$ where the tangent line is $\parallel$ to the secant line that cuts the curve @ $x=-1$ & $x=2$

$(-1, f(-1)) = (-1, 1)$

$(2, f(2)) = (2, 4)$

$m = \frac{4-1}{2-(-1)} = \frac{3}{3} = 1$

$\therefore m_{\text{tan line}} = 1$

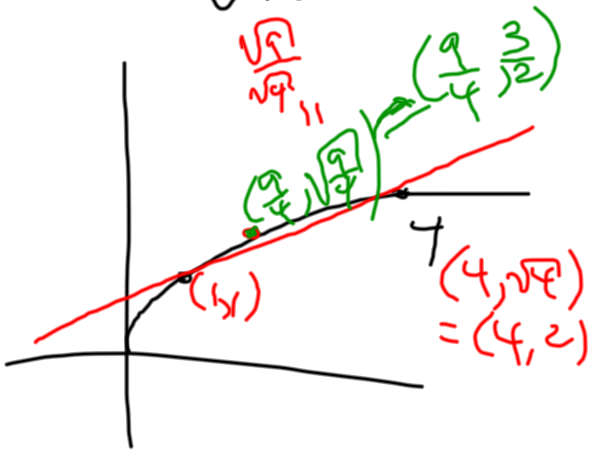
slope = value of deriv.

$y' = 2x = 1$

solve: $x = \frac{1}{2}$

60) Find the x-coord of . . . . $y = \sqrt{x}$

when . . . . $x = 1$, and $x = 4$



$m_{line} = \frac{2-1}{4-1} = \frac{1}{3}$

$\therefore m_{\text{tan line}} = \frac{1}{3}$

slope = value of deriv.

$$y' = \frac{d}{dx}\left(x^{1/2}\right) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$\frac{1}{2\sqrt{x}} = \frac{1}{3} \Rightarrow 3 = 2\sqrt{x}$$

$$\frac{3}{2} = \sqrt{x}$$

$$\frac{9}{4} = x$$

Find eqn of tan line to $y = x^{1/3}$ @ $x = 0$.

slope $= f'(x)$

$\text{slope}\big|_{x=0} = f'(0)$

$f'(x) = \frac{1}{3} x^{-2/3} = (x^{1/3})^{-2}$

$= \frac{1}{3} \frac{1}{(\sqrt[3]{x})^2}$

$f'(0) =$ undefined

Pt: $(0, f(0))$

$= (0, 0^{1/3}) = (0,0)$

$x = 0$

6) $y = \sqrt{2}\,x + \left(\frac{1}{\sqrt{2}}\right)$

$$\frac{dy}{dx} = \sqrt{2}\,\frac{d}{dx}(x^1) + \frac{d}{dx}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \sqrt{2}\,(1) + 0$$

$$= \sqrt{2}$$

$\frac{1}{\sqrt{2}}x^0$

$\frac{1}{\sqrt{2}}(0x^{-1})$

8

$$\frac{d}{dx}\left(\sqrt{2x}\right) = \sqrt{2}\,\frac{d}{dx}\left(\sqrt{x}\right)$$

$$= \sqrt{2}\,\frac{d}{dx}\left(x^{1/2}\right)$$

$$= \sqrt{2}\left(\frac{1}{2}x^{-1/2}\right) = \frac{\sqrt{2}}{2\sqrt{x}}$$

8) $y = \frac{x^2+1}{5} = \frac{x^2}{5} + \frac{1}{5}$

$\frac{dy}{dx} = \frac{1}{5}\frac{d}{dx}(x^2) + \frac{d}{dx}\left(\frac{1}{5}\right)$

$= \frac{1}{5}(2x) + 0$

$= \frac{2}{5}x$

$y = \frac{1}{5}$

Derivative

slope of tan line

inst rate of chg

take as many as you want

$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

12) $y = 7x^{-6} - 5\sqrt{x} = 7x^{-6} - 5x^{1/2}$

$$\frac{dy}{dx} = 7\left(-6x^{-7}\right) - 5\left(\frac{1}{2}x^{-1/2}\right)$$

$$= -42x^{-7} - \frac{5}{2}x^{-1/2}$$

$$= -42x^{-7} - \frac{5}{2\sqrt{x}} = \frac{-42}{x^7} - \frac{5}{2\sqrt{x}}$$

11) $y = -3x^{-8} + 2\sqrt{x}$

$$\frac{dy}{dx} = -3\frac{d}{dx}\left(x^{-8}\right) + 2\frac{d}{dx}\left(x^{1/2}\right)$$

$$= -3\left(-8x^{-9}\right) + 2\left(\tfrac{1}{2}x^{-1/2}\right)$$

$$= 24x^{-9} + x^{-1/2} \ldots$$

Frac

$x^{-\frac{1}{2}}$ — $\sqrt{\ }$

$$= \frac{1}{x^{1/2}} = \frac{1}{\sqrt{x}}$$

$\underline{16)}$ $f(x) = (2 - x - 3x^3)(7 + x^5)$

17
21
16 19
18

$f'(x) = \quad f'g + fg'$

$= (-1 - 9x^2)(7 + x^5) + (2 - x - 3x^3)(5x^4)$

$= -7 - 63x^2 - x^5 - 9x^7 +$

$10x^4 - 5x^5 - 15x^7$

$-24x^7 - 6x^5 + 10x^4 - 63x^2 - 7$

17) $f(x) = (x^3+7x^2-8)(2x^{-3}+x^{-4})$

$f'(x) = (3x^2+14x)(2x^{-3}+x^{-4}) + (x^3+7x^2-8)(-6x^{-4}-4x^{-5})$

$= 6x^{-1}+28x^{-2}+3x^{-2}+14x^{-3}+$

$-6x^{-1}-4x^{-2}-42x^{-2}-28x^{-3}+48x^{-4}+32x^{-5}$

$= -15x^{-2}-14x^{-3}+48x^{-4}+32x^{-5}$

$\frac{d}{dx}(x^n) = nx^{n-1}$
mult by exp
sub 1 from exp

ANT
add 1 to exp
divide by new exp

$\int x^n dx = \frac{x^{n+1}}{n+1}$

$x^{-1} + x^{-2}$

$\boxed{18}$ $f(x) = \left(\frac{1}{x} + \frac{1}{x^2}\right)(3x^3 + 27)$

$f'(x) = (-x^{-2} - 2x^{-3})(3x^3 + 27) + \left(\frac{1}{x} + \frac{1}{x^2}\right)(9x^2)$

$= -3x - 27x^{-2} - 6 - 54x^{-3} + 9x + 9$

$= 6x + 3 - 27x^{-2} - 54x^{-3}$

19) $f(x) = (3x^2+1)^2 = (3x^2+1)(3x^2+1)$

$f'(x) = (6x)(3x^2+1) + (3x^2+1)(6x)$

$= 2(6x)(3x^2+1) = 36x^3 + 12x$

21) $f(x) = \dfrac{1}{(5x-3)}$ $1 \rightarrow f$, $(5x-3) \rightarrow g$

$$f'(x) = \frac{\frac{d}{dx}(1)\cdot(5x-3) - (1)\frac{d}{dx}(5x-3)}{(5x-3)^2}$$

$$= \frac{0(\sim) - 5}{(5x-3)^2}$$

$$= \frac{-5}{(5x-3)^2}$$

$$f'(1) = \frac{-5}{(5(1)-3)^2}$$

$$= \frac{-5}{(2)^2}$$

$$= \frac{-5}{4}$$

22) $y = \dfrac{3}{\sqrt{x}+2}$

$\frac{d}{dx}(x^{1/2}+2) = \frac{1}{2}x^{-\frac{1}{2}}$

$$y' = \frac{(0)(\sqrt{x}+2) - (3)(\frac{1}{2}x^{-1/2})}{(\sqrt{x}+2)^2} = \frac{\frac{-3}{2\sqrt{x}}}{(\sqrt{x}+2)^2}\left(\frac{1}{(\sqrt{x}+2)^2}\right)\Big/\left(\frac{1}{(\sqrt{x}+2)^2}\right)$$

$$\frac{-3}{2\sqrt{x}(\sqrt{x}+2)^2}$$

STOP

22 b. $(f'(1)) =$

$$\frac{-3}{2\sqrt{1}(\sqrt{1}+2)^2} = \frac{-3}{2(3^2)}$$

$$= \frac{-1}{6}$$

26, 27-28
34

26) $y = \frac{(4x+1)}{(x^2-5)}$ Find $\left.\frac{dy}{dx}\right|_{x=1}$

$$y' = \frac{f'g - fg'}{(g)^2} = \frac{\frac{d}{dx}(4x+1)(x^2-5) - (4x+1)\frac{d}{dx}(x^2-5)}{(x^2-5)^2}$$

$$= \frac{(4)(x^2-5)-(4x+1)(2x)}{(x^2-5)^2}$$

$$y'(x=1) = \frac{(4)(1^2-5)-(4(1)+1)(2(1))}{(1^2-5)^2} = \frac{(4)(-4)-(5)(2)}{16}$$

$$= \frac{-16-10}{16} = \frac{-26}{16} = \frac{-13}{8}$$

27) $y=\left(\frac{3x+2}{x}\right)\left(x^{-5}+1\right)$

$(fg)'=f'g+fg'$

$\frac{d}{dx}\left(\frac{3x}{x}+\frac{2}{x}\right)=\underbrace{\frac{d}{dx}(3)+\frac{d}{dx}(2x^{-1})}$

$f=\left(\frac{3x+2}{x}\right)$; $f'=\frac{d}{dx}\left(\frac{3x+2}{x}\right)=\frac{(3)(x)-(3x+2)(1)}{(x)^2}=\frac{-2}{x^2}$

$t=3x+2$; $t'=3$

$b=x$; $b'=1$

$g=\left(x^{-5}+1\right)$; $g'=-5x^{-6}$

$$y'=\left(-\frac{2}{x^2}\right)\left(x^{-5}+1\right)+\left(\frac{3x+2}{x}\right)\left(-5x^{-6}\right)$$

$$y'(1)=\left(-\frac{2}{1}\right)(2)+\left(\frac{5}{1}\right)(-5)=-4-25=-29$$

28) $y=(2x^7-x^2)\left(\frac{x-1}{x+1}\right)$

$$y'=(14x^6-2x)\left(\frac{x-1}{x+1}\right)+(2x^7-x^2)\left(\frac{(1)(x+1)-(x-1)(1)}{(x+1)^2}\right)$$

$$y'(1)=(12)(0)+(1)\left(\frac{2-0}{2^2}\right)=\frac{1}{2}$$

34) $\frac{dC}{dr}$ where $C=2\pi r$

$$\frac{dC}{dr}=\frac{d}{dr}(2\pi r)=2\pi\frac{d}{dr}(r)=2\pi(1)$$

$$=2\pi$$

39
43-46
57-60

39) Find $g'(4)$ given $f(4)=3$ and $f'(4)=-5$.

a) $g(x)=\sqrt{x}\,f(x)$

$$g'(x) = (\text{first})'(\text{second}) + (\text{first})(\text{second})'$$

$\sqrt{x} = x^{1/2}$

$\frac{d}{dx}(x^{1/2})$

$= \frac{1}{2}x^{-1/2}$

$= \frac{1}{2\sqrt{x}}$

$$= \left(\frac{1}{2\sqrt{x}}\right)\left(f(x)\right) + \left(\sqrt{x}\right)\left(f'(x)\right)$$

$$g'(4) = \left(\frac{1}{2\sqrt{4}}\right)\left(f(4)\right) + \left(\sqrt{4}\right)\left(f'(4)\right)$$

$$= \left(\frac{1}{4}\right)(3) + (2)(-5) = \frac{3}{4} - 10 = \frac{-37}{4}$$

b) $g(x) = \frac{f(x)}{x}$

$$g'(x) = \frac{(\text{Top})'(\text{Bottom}) - (\text{Top})(\text{Bottom})'}{(x)^2} = \frac{(f'(x))(x) - (f(x))(1)}{(x)^2}$$

$$g'(4) = \frac{f'(4)\cdot(4) - f(4)}{4^2} = \frac{(-5)(4)-(3)}{16} = \frac{-23}{16}$$

41) Find $F'(2)$ given $f(2) = -1$, $g(2) = 1$, $f'(2) = 4$, $g'(2) = -5$

a) $F(x) = 5f(x) + 2g(x)$

$F'(x) = 5f'(x) + 2g'(x)$

$\left[f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \right]$

$F'(2) = 5(4) + 2(-5)$

$= 20 + (-10) = 10$

b) $F(x) = f(x) - 3g(x)$

$F'(x) = f'(x) - 3g'(x)$

$F'(2) = f'(2) - 3g'(2)$

$= 4 - 3(-5)$

$= 4 + 15 = 19$

43) Find an equation of the tangent to the graph of $y=f(x)$ at $x=-3$ if $f(-3)=2$ and $f'(-3)=5$.

To write the equation of a line.... I NEED

the slope — a point

slope = value of of the derivative

$= f'(-3) = 5$

pt $= (x, y)$

$= (-3, f(-3))$

$= (-3, 2)$

$\therefore$ Eqn:

$$y-2=5(x-(-3))$$

45) find $\frac{d^2y}{dx^2}$ (second derivative)

(a) $y = 7x^3 - 5x^2 + x$

$7\frac{d}{dx}(x^3)$

$y' = 21x^2 - 10x + 1$

$y'' = (y')' = 42x - 10$

(b) $y = 12x^2 - 2x + 3$

$y' = 24x - 2$

$y'' = (y')' = 24$

c) $y = \frac{x+1}{x} = 1 + \frac{1}{x} = 1 + x^{-1}$

$y' = \frac{d}{dx}(1) + \frac{d}{dx}(x^{-1})$

$= -x^{-2}$

$y'' = (y')' = +2x^{-3}$

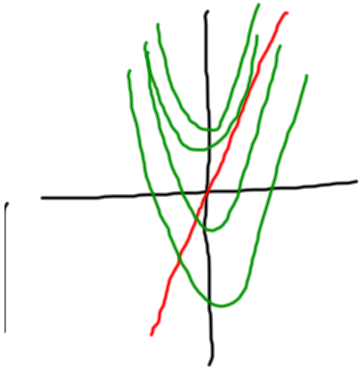
(d) $y = (5x^2-3)(7x^3+x)$

$y' = (10x)(7x^3+x) + (5x^2-3)(21x^2+$

$y'' = (y')' = (10)(7x^3+x) + (10x)(21x^2+1)$

$+ (10x)(21x^2+1) + (5x^2-3)(42$

58) Find k if the curve $y=x^2+k$ is tangent to the line $y=2x$



1) Assume that the pt of ~~intersection~~ tangency $=(a,b)$

2) slope of $y=2x$ $=$ *2*

3) derivative of $y=x^2+k$ [which rep slope]

$= y'=2x=2$

$\therefore x=1$

in other words, $a=1$

4) pt of tangency $=(1,b)$

now $b=(1)^2+k$ on the curve

& $b=2(1)=2$ on the line

So $(\therefore)$ $1^2+k=2$

$\Rightarrow k=1$

Chk graph

$Y_1=x^2+1$

$Y_2=2x$

$$f(x) = \begin{cases} x^2+x+1, & x \le 1 \\ mx+b, & x > 1 \end{cases}$$

Find values of m, and b, that make f(x) continuous @ x=1 & differentiable @ x=1

Cont

$$\lim_{x\to 1^-} x^2+x+1 = \lim_{x\to 1^+} mx+b$$

$$\underbrace{x^2+x+1}_{=\,3} \qquad \underbrace{mx+b}_{=\,m+b}$$

$3 = 3+b$

$\Rightarrow b = 0$

diff

$$f'(x) = \begin{cases} 2x+1, & x<1 \\ m, & x>1 \end{cases}$$

f'(1) "from the left"

$= 2(1)+1 = 3$

f'(1) "from the right"

$= m$

$\therefore \boxed{m=3}$

78) $f(x) = \begin{cases} x^3 + \frac{1}{16}, & x < \frac{1}{2} \\ \frac{3}{4}x^2, & x \geq \frac{1}{2} \end{cases}$

$\lim_{x \to \frac{1}{2}^-} x^3 + \frac{1}{16} = \frac{3}{16}$

$\lim_{x \to \frac{1}{2}^+} \frac{3}{4}x^2 = \frac{3}{16}$

Q: what does it mean for a piecewise f^n to be differentiable?

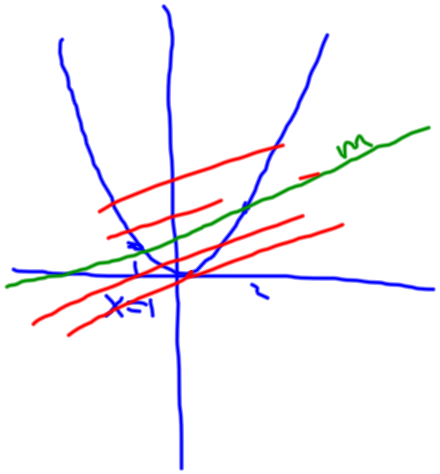
$f'(x) = \begin{cases} 3x^2, & x < \frac{1}{2} \\ \frac{3}{2}x, & x > \frac{1}{2} \end{cases}$

from both sides

$3\left(\frac{1}{2}\right)^2 \overset{?}{=} \frac{3}{2}\left(\frac{1}{2}\right) \quad \therefore$ YES

$= \frac{3}{4} \qquad = \frac{3}{4}$

59) Find the x-coord of the pt on the graph of $y=x^2$ ($f(x)$) where the tangent line is || the secant line cuts thru curve @ $x=-1$, & $x=2$



? what is slope of secant line?

$$\frac{f(2)-f(-1)}{2-(-1)} = \frac{4-(1)}{2-(-1)} = \frac{3}{3} = 1$$

? what is the slope of the tangent line? $= 1$

? what x-value has a "slope" of 1

i.e. where is derivative value $=1$?

$$y=x^2 \Rightarrow y'=2x \qquad 2x=1 \Rightarrow x=\tfrac{1}{2}$$

If they wanted the point, then $y=\left(\frac{1}{2}\right)^2=\frac{1}{4}$

3.3 homework

2010-10-07 Pd 3